

Interaction of E.M.W. with Matter  
on Macroscopic scale.

To Prove Law of Reflection and Refraction.

The equation (1) can only be satisfied if the time and space varying components of the phase in equation (1) we get

$$\omega_i t + \omega_R t = \omega_T t$$

$$\omega_i = \omega_R = \omega_T = \omega \quad \text{————— (2)}$$

The eqn (2) shows that the frequency of the wave remains unchanged by reflection and refraction.

And equating the space varying components of the phase in equation (1) we get

$$(k_i \cdot r)_{z=0} = (k_R \cdot r)_{z=0} = (k_T \cdot r)_{z=0} \quad \text{————— (3)}$$

Now as the incident beam is in  $x-z$  plane  $n_y$  is zero. It then follows that  $y$  term in the other expression of eqn (3) are also zero, i.e. the reflected and transmitted wave are in the same plane as the incident ray and the normal ( $z$ -axis).

Further in the light of above, for incident, reflected and transmitted wave we have

$$(k_i \cdot r) = k, (x \sin \theta_i + z \cos \theta_i) \quad \text{————— (4)}$$

$$(k_R \cdot r) = k_R (x \sin \theta_R - z \cos \theta_R) \quad \text{--- (5)}$$

$$(k_r \cdot r) = k_r (x \sin \theta_r + z \cos \theta_r) \quad \text{--- (6)}$$

In the light of eqn (3), (4) and (5) yield

$$k_i \sin \theta_i = k_R \sin \theta_R$$

but as  $\omega_i = \omega_R$  and  $v_i = v_R$  as the medium is same

$$k_i = k_R$$

$$\text{So } \sin \theta_i = \sin \theta_R \quad \text{ie } \theta_i = \theta_R \quad \text{--- (7)}$$

And in the light of equation (3), (4) and (6) yield

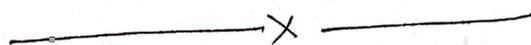
$$k_i \sin \theta_i = k_r \sin \theta_r$$

$$\text{ie } \frac{\omega_i}{v_i} \sin \theta_i = \frac{\omega}{v_r} \sin \theta_r \quad \left[ \text{as } k = \frac{\omega}{v} \right]$$

$$\text{ie } \frac{c}{v_i} \sin \theta_i = \frac{c}{v_r} \sin \theta_r \quad \left[ \text{dividing by } \frac{c}{\omega} \right]$$

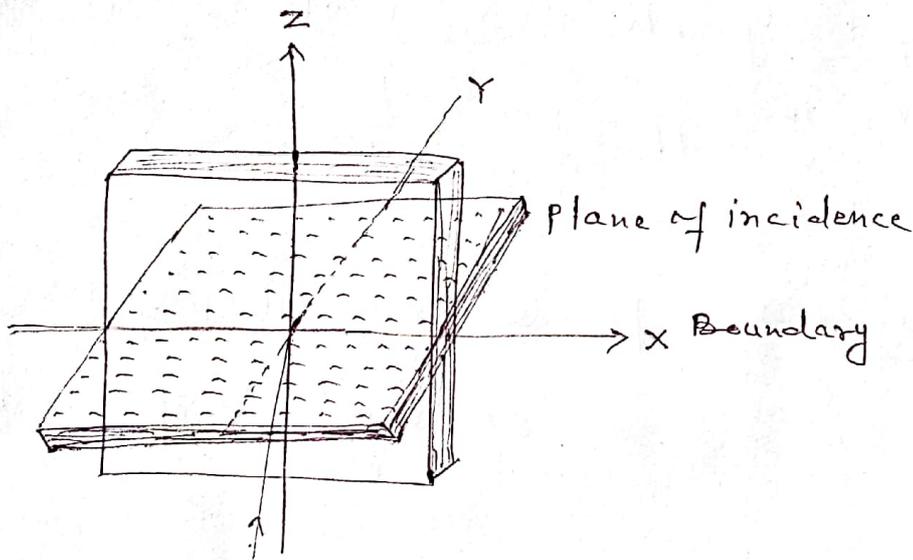
$$\text{ie } n_1 \sin \theta_i = n_2 \sin \theta_r \quad \text{--- (8)}$$

Eqn (7) and (8) are desired result,



## Ferisnel's formulae (Dynamic properties)

The formulae relating the amplitudes of the reflected and transmitted electromagnetic waves with that of incident one when the boundary is between two dielectrics are called Ferisnel's formulae. These are contained in the boundary condition i.e. in



$$(D_i)_n + (D_R)_n = (D_T)_n \quad \text{--- (1)}$$

$$(B_i)_n + (B_R)_n = (B_T)_n \quad \text{--- (2)}$$

$$(E_i)_t + (E_R)_t = (E_T)_t \quad \text{--- (3)}$$

$$\text{and } (H_i)_t + (H_R)_t = (H_T)_t \quad \text{--- (4)}$$

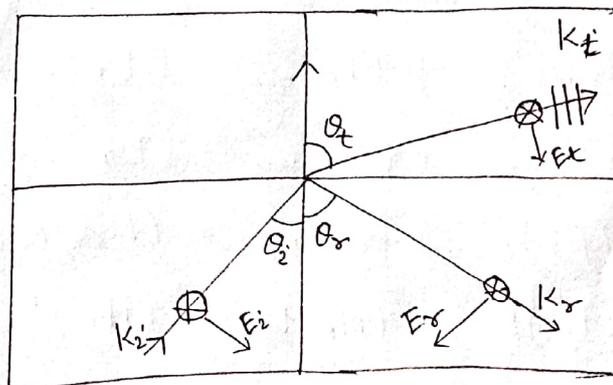
When eqn (1) and (2) coupled with Snell's Law yield no information not included in the equation (3) and (4). So it is necessary to consider only eqn (3) and (4),

Now to derive the desired formulae

we consider a plane E.M.W. in  $x-z$  plane incident on a plane boundary and consider it as a superposition of two waves one with the electric vector parallel to the plane of incidence and the other with electric vector parallel to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.

Case 1.  $E$  - parallel to the plane of incidence.

The situation is shown in figure below.



The electric and propagation vectors in two media are indicated. The directions of  $H$  vectors are chosen so as to give a positive flow of energy in the direction of wave vectors.

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